

ECE 631 System Theory - Problem Set 3

(due on Tuesday, 17 March 2020 at 18:00)

PROBLEM 1

(15 points)

Check whether the function $f : \mathbb{R}^2 \mapsto \mathbb{R}$ given by $f(x) = f\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = (x_1^2 + 2x_1x_2 + 4x_2^2)^{1/2}$ defines a norm on \mathbb{R}^2 .

PROBLEM 2

(10 points)

Compute $\|A\|_1, \|A\|_2, \|A\|_\infty$ for the matrices

$$(a) \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}, \quad (b) \begin{bmatrix} 1 & 2 \\ 2 & 0 \end{bmatrix}, \quad (c) \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, \quad (d) \begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix}$$

PROBLEM 3

(10 points)

Sketch the set $\mathcal{U} = \{x \in \mathbb{R}^2 \mid \|x - c\| = \|c\|_u\}$, where $c = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$, for the three cases where u is the 1, 2, and ∞ norm. Sketch all three on the same diagram.

PROBLEM 4

(15 points)

Consider the linear space $C[0,5]$ (that is, the set of continuous functions defined on the interval $0 \leq t \leq 5$). Compute $\|x\|_1, \|x\|_2, \|x\|_\infty$ for the function $e^{t/3} - t$ (you may use MATLAB to compute any zero-crossings).

PROBLEM 5

(15 points)

Find the point on the line $3x + 2y = 5$ in two dimensional space closest to the origin when distance is measured by each of the following two norms: (a) the 1-norm, and (b) the ∞ -norm.

PROBLEM 6

(20 points)

Find the SVD of A and B , where $A = \begin{bmatrix} 4 & 11 & 14 \\ 8 & 7 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$.

MATLAB ASSIGNMENT

(15 points)

This exercise is intended to familiarize you with some matrix manipulation functions of MATLAB.

Using File – > New – > M-File, create a file in the same directory as your MATLAB environment and name it “*filehw2.m*” ; this is called an M-File. Write the following in “*filehw2.m*”:

```
for i = 1:7
for j = 1:7
A(i, j) = (4*rand)-2 ;
end
end
A
```

This will create a matrix of dimension 7 by 7 with random entries between -2 and 2. Now, type:

```
>> filehw2
```

Calculate the following of your matrix *A*:

(a) determinant; (b) eigenvalues, eigenvectors ; (c) condition number; (d) rank; (e) inverse; (f) trace; (g) characteristic polynomial; (h) norm; (i) null space; (j) singular value decomposition.

Create a “diary” of your MATLAB session and submit it with your homework. (try “*help diary*”). Do “*help*” on the following MATLAB commands to obtain on-line information on how to calculate the above quantities: “*det*”, “*eig*”, “*cond*”, “*rank*”, “*inv*”, “*trace*”, “*poly*”, “*norm*”, “*null*”, “*svd*”. (do not include these “help” commands in the diary that you turn in.)