University of Cyprus
Department of Electrical and Computer Engineering

ECE 631 System Theory - Problem Set 4

(due on Tuesday, 5 April 2020 at 24:00)

Problem 1 (20 points)

Consider the nonlinear state equation

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} u(t) \\ x_1(t)u(t) - x_3(t) \\ x_2(t) - 2x_3(t) \end{bmatrix}$$

$$y(t) = x_2(t) - 2x_3(t)$$

with the nominal initial state $\bar{x}_1(0) = 0$, $\bar{x}_2(0) = -3$, $\bar{x}_3(0) = -2$, and the nominal input $\bar{u}(t) = 1$. Show that the nominal output is $\bar{y}(t) = 1$. Linearise the state equation about the nominal solution.

Problem 2 (20 points)

Consider the nth-order linear differential equation

$$y^{(n)}(t) + \alpha_{n-1}(t)y^{n-1}(t) + \ldots + \alpha_0(t)y(t) = b_0(t)u(t) + b_1(t)u^{(1)}(t)$$

Draw a simulation diagram and rewrite the above differential equation as a n-dimensional linear state equation

$$\dot{x}(t) = A(t)x(t) + B(t)u(t)$$
$$y(t) = C(t)x(t) + D(t)u(t)$$

(Hint: Let $x_n(t) = y^{(n-1)}(t) - b_1(t)u(t)$.)

Problem 3 (20 points)

The Euler equations for the angular velocities of a rigid body are important in the modelling of robotic manipulators. The Euler equations are given by

$$I_1\dot{\omega}_1(t) = (I_2 - I_3)\omega_2(t)\omega_3(t) + u_1(t)$$
$$I_2\dot{\omega}_2(t) = (I_3 - I_1)\omega_1(t)\omega_3(t) + u_2(t)$$
$$I_3\dot{\omega}_3(t) = (I_1 - I_2)\omega_1(t)\omega_2(t) + u_3(t)$$

Here $\omega_1(t)$, $\omega_2(t)$, and, $\omega_3(t)$ are the angular velocities in a body-fixed coordinate system coinciding with the principal axes, $u_1(t)$, $u_2(t)$, and $u_3(t)$ are the applied torques, and I_1, I_2, I_3 are the principal moments of inertia. For $I_1 = I_2$ (a symmetrical body), linearise the equations about the nominal solution

$$\bar{u}_1(t) = \bar{u}_2(t) = \bar{u}_3(t) = 0, \quad \bar{\omega}_1(0) = 0, \quad \bar{\omega}_2(0) = 1, \quad \bar{\omega}_3(0) = \omega_0$$

$$\bar{\omega}_1(t) = \sin\left(\omega_0 \frac{I - I_3}{I}\right), \quad \bar{\omega}_2(t) = \cos\left(\omega_0 \frac{I - I_3}{I}\right), \quad \bar{\omega}_3(t) = \omega_0$$

where $I = I_1 = I_2$.

MATLAB ASSIGNMENT (simulate a satellite) (40 points)

Write a MATLAB program to solve the nonlinear differential equation that describes the satellite motion as given in the class notes. Assume that: $\beta = 10$; $x_1(0) = r_0 = 10$; $x_2(0) = \dot{r}(0) = 0$; $x_3(0) = \theta_0 = 0$. Consider the following cases:

(a) $x_4(0) = 0.1, \quad u_1(t) = 0, \quad u_2(t) = 0.$

(b) $x_4(0) = 0.095, \quad u_1(t) = 0, \quad u_2(t) = 0.$

(c) $x_4(0) = 0.105, \quad u_1(t) = 0, \quad u_2(t) = 0.$

(d) $x_4(0) = 0.1, \quad u_1(t) = 0.02, \quad u_2(t) = 0.$

(e) $x_4(0) = 0.1, \quad u_1(t) = 0.1\sin(t), \quad u_2(t) = 0.1\cos(t)\,.$

(f) $x_4(0) = 0.09, \quad u_1(t) = 0, \quad u_2(t) = 0.1\cos(t)\,.$

Simulate the differential equation for about 100 seconds (i.e., $t_f = 100$). Provide plots of the satellite motion in cartesian coordinates instead of polar coordinates. Interpret your results. Compare the solution of the nonlinear differential equation with that of the linearised model (assume that $r_0 = 10$; $\theta_0 = 0$; $\omega = 0.1$). Discuss the accuracy of the linearised model as an approximation of the nonlinear system. Plot the trajectories of the satellite motion of both the linear and nonlinear model on the same diagram for comparison purposes.