University of Cyprus
Department of Electrical and Computer Engineering

ECE 631 System Theory - Problem Set 5

(due on Friday, 24 April 2020 at 24:00)

Problem 1 (20 points)

(a) An oscillation can be generated by

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} x$$

Show that its solution is

$$x(t) = \begin{bmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{bmatrix} x(0)$$

(b) Show

$$\frac{\partial}{\partial t}\Phi(t_0, t) = -\Phi(t_0, t)A(t)$$

Problem 2 (20 points)

Find fundamental matrices and state transition matrices for

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & t \end{bmatrix} x$$

and for

$$\dot{x} = \begin{bmatrix} -1 & e^{2t} \\ 0 & -1 \end{bmatrix} x.$$

Problem 3 (20 points)

Consider the 2-input, 3-output, 2-dimensional system given by

$$\dot{x} = \begin{bmatrix} -3 & 1 \\ -2 & -1 \end{bmatrix} x + \begin{bmatrix} 3 & 2 \\ -2 & 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 2 \\ 2 & 2 \\ 1 & -1 \end{bmatrix} x$$

Compute the response of the system $y(t)$ for $u(t) = 0$ and for initial condition $x(0) = [1 \;\; 1]^T$.

PROBLEM 4 (20 points)

A linearised model of a satellite (not the same one described in class) has the form $\dot{x}(t) = Ax(t)$, where A is a 4×4 matrix. Given that the state transition matrix for this system is

$$\Phi(t,0) = e^{At} = \begin{bmatrix} 4\cos(\omega t) - 3 & 0 & 6\omega(-1+\cos(\omega t)) & -2\sin(\omega t) \\ \frac{1}{\omega}(4\sin(\omega t) - 3\omega t) & 1 & 6(-\omega t + \sin(\omega t)) & \frac{-2}{\omega}(1-\cos(\omega t)) \\ \frac{2}{\omega}(1-\cos(\omega t)) & 0 & 4 - 3\cos(\omega t) & \frac{1}{\omega}\sin(\omega t) \\ 2\sin(\omega t) & 0 & 3\omega\sin(\omega t) & \cos(\omega t) \end{bmatrix}$$

find the matrix A. In general, if you are given the state transition matrix $\Phi(t, t_0)$ of a linear, time-varying system $\dot{x} = A(t)x(t)$, how can you find $A(t)$? [Hint. you do not need to invert any matrix]

PROBLEM 5 (20 points)

Consider a linear time-invariant SISO system with state variable realization $\{A, B, C, D\}$; i.e.,

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t)$$

Assume $D \neq 0$. As shown in class, the transfer function of this system is given by

$$G(s) = C(sI - A)^{-1}B + D\,.$$

Now consider the system with transfer function $1/G(s)$. Show that a state variable realization of this system is given by $\{\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D}\}$, where

$$\tilde{A} = A - (BC/D)$$
$$\tilde{B} = B/D$$
$$\tilde{C} = -C/D$$
$$\tilde{D} = 1/D$$