University of Cyprus
Department of Electrical and Computer Engineering

ECE 631 System Theory - Problem Set 6

(due on Thursday, 30 April 2020 at 24:00)

PROBLEM 1 (25 points)

Consider the system given by

$$\dot{x} = \begin{bmatrix} -2 & 0 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$

$$y = [-1 \quad 2]\, x$$

(a) Find all equilibrium solutions x_e.

(b) Determine which equilibria are asymptotically stable.

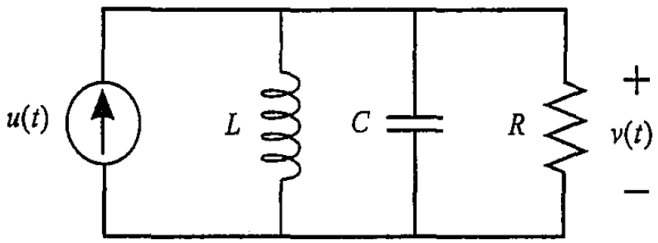
(c) Determine if the equilibrium solutions are Lyapunov stable.

(d) Determine if the system is BIBO stable.

(e) Let $z_1 = x_1$, $z_2 = -x_1 + x_2$, and $u(t) = 0$. If we denote $z \triangleq [z_1 \ z_2]^T$ and $\dot{z} = \hat{A} z$, find the equilibrium solutions z_e.

PROBLEM 2 (25 points)

Determine whether the following system is BIBO stable. Consider the input to be the current $u(t)$, and the output to be the voltage $v(t)$. Repeat if $R = \infty$ (i.e., remove the resistor).



PROBLEM 3 (25 points)

Determine whether each of the systems below is controllable and/or observable

(a) $$\dot{x} = \begin{bmatrix} -7 & -2 & 6 \\ 2 & -3 & -2 \\ -2 & -2 & 1 \end{bmatrix} x + \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 1 & 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} -1 & -1 & 2 \\ 1 & 1 & -1 \end{bmatrix} x$$

(b) $A = \begin{bmatrix} 2 & -5 \\ -4 & 0 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad c = [1 \quad 1]$

(c) $\dot{x} = \begin{bmatrix} 0 & 1 \\ -4 & -4 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$

$y = [1 \quad 1]\, x$

PROBLEM 4 (25 points)

For the electrical circuit shown below, find conditions on C_1 and C_2 that will make the system uncontrollable. Consider v_g to be the input, and v_1 and v_2 to be the state variables.