ECE 631 – HOMEWORK #6 – SOLUTIONS.

1

7.15 Consider the system given by

$$\dot{x} = \begin{bmatrix} -2 & 0 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$
$$y = \begin{bmatrix} -1 & 2 \end{bmatrix} x$$

a) Find all equilibrium solutions x_e.

Solution:

$Ax_e = 0$: equilibrium points are x_e, such that $x_e \in N(A)$, or $x_e = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ or ANY SCALAR MULTIPLE.

b) Determine which equilibria are asymptotically stable.

Solution:

Since one eigenvalue is zero, none of the equilibria are ASYMPTOTICALLY stable.

c) Determine if the equilibrium solutions are Lyapunov stable.

Solution:

A has a non-positive real eigenvalue and non-repeated zero eigenvalue. So, the equilibria are Lyapunov stable.

d) Determine if the system is BIBO stable.

Solution:

For BIBO stability, find transfer function. [NUM,DEN]=ss2tf(A,B,C,D)

$\Rightarrow \dfrac{s+4}{s(s+2)}$.

POLE on IMAGINARY AXIS ($s=0$) => NOT BIBO stable.

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e) Let $z_1 = x_1$, and $z_2 = -x_1 + x_2$. Also let $u(t) = 0$. If we denote $z \triangleq [z_1 \quad z_2]^T$ and $\dot{z} = \hat{A}z$, find the equilibrium solutions z_e, and sketch them on the $z_1 - z_2$ plane.

Solution:

Let $z = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} x \triangleq Px$ or $x = P^{-1}z$.

Then,

$$\dot{z} = P\dot{x} = PAx = PAP^{-1}z$$
$$\hat{A} = PAP^{-1} = \begin{bmatrix} -2 & 0 \\ 2 & 0 \end{bmatrix}.$$

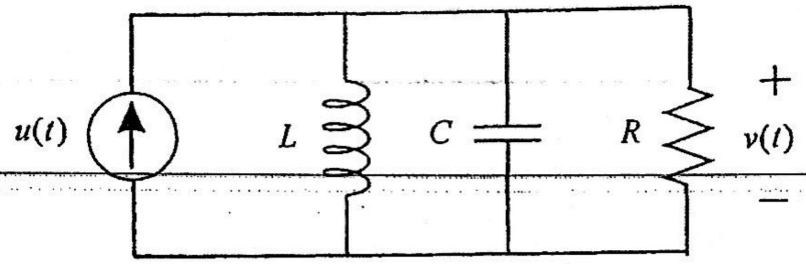
So,

$$\dot{z} = \begin{bmatrix} -2 & 0 \\ 2 & 0 \end{bmatrix} z.$$

Equilibrium $z_e = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

2

7.13 Determine whether or not the system shown in the following diagram is BIBO stable. Consider the input to be current $u(t)$, and output to be voltage $v(t)$.



P7.13

Repeat if $R = \infty$ (i.e., remove resistor).

Solution:

i)

$$\frac{v(t)}{R} + C\frac{dv(t)}{dt} + \frac{1}{L}\int v(t)dt = u(t)$$

$$\frac{1}{R}\dot{v}(t) + C\ddot{v}(t) + \frac{1}{L}v(t) = \dot{u}(t) \quad \Rightarrow \quad G(s) = \frac{\frac{1}{C}s}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}$$

The system poles has negative real parts at $-\frac{1}{2RC}$. Therefore, the system is BIBO stable.

ii) When $R = \infty$,

$$C\frac{dv(t)}{dt} + \frac{1}{L}\int v(t)dt = u(t)$$

$$C\ddot{v}(t) + \frac{1}{L}v(t) = \dot{u}(t) \quad \Rightarrow \quad G(s) = \frac{\frac{1}{C}s}{s^2 + \frac{1}{LC}}$$

The poles are at $\pm j\frac{1}{\sqrt{LC}}$. So, NOT BIBO stable.

3

8.1 Determine whether each of the systems below is controllable and/or observable

b)

$$\dot{x} = \begin{bmatrix} -7 & -2 & 6 \\ 2 & -3 & -2 \\ -2 & -2 & 1 \end{bmatrix} x + \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 1 & 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} -1 & -1 & 2 \\ 1 & 1 & -1 \end{bmatrix} x$$

Solution:

rank(ctrb(A, B)) = 2 => NOT controllable

rank(obsv(A, B)) = 2 => NOT observable

c) $A = \begin{bmatrix} 2 & -5 \\ -4 & 0 \end{bmatrix}$ $b = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ $c = \begin{bmatrix} 1 & 1 \end{bmatrix}$

Solution:

rank(ctrb(A, B)) = 2 => controllable

rank(obsv(A, B)) = 2 => observable

d)

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -4 & -4 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 1 \end{bmatrix} x$$

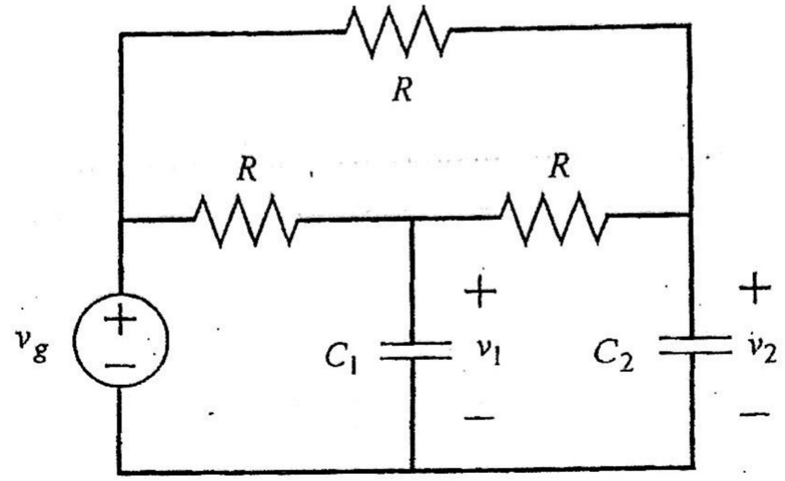
Solution:

rank(ctrb(A, B)) = 2 => controllable

rank(obsv(A, B)) = 2 => observable

4

8.8 For the electrical circuit shown below, find conditions on C_1 and C_2 that will make the system uncontrollable. Consider v_g to be the input, and v_1 and v_2 to be the state variables.



P8.8

Solution: $C_1 = C_2$

Using the KCL,

$$\frac{v_1 - v_2}{R} + \frac{v_1 - v_g}{R} = C_1\frac{dv_1}{dt} \qquad \frac{v_2 - v_1}{R} + \frac{v_2 - v_g}{R} = C_2\frac{dv_2}{dt}$$

$$\dot{v}_1 = \frac{2}{C_1R}v_1 - \frac{1}{C_1R}v_2 - \frac{1}{C_1R}v_g \qquad \dot{v}_2 = -\frac{1}{C_2R}v_1 + \frac{2}{C_2R}v_2 - \frac{1}{C_2R}v_g$$

$$\dot{v} = \begin{bmatrix} \frac{2}{C_1R} & -\frac{1}{C_1R} \\ -\frac{1}{C_2R} & \frac{2}{C_2R} \end{bmatrix} v + \begin{bmatrix} -\frac{1}{C_1R} \\ -\frac{1}{C_2R} \end{bmatrix} v_g \qquad P = \begin{bmatrix} -\frac{1}{C_1R} & -\frac{2}{C_1^2R^2} + \frac{1}{C_1C_2R} \\ -\frac{1}{C_2R} & \frac{1}{C_1C_2R} - \frac{2}{C_2^2R^2} \end{bmatrix}$$

For the system to be uncontrollable, matrix P must be rank degenerate; i.e., $\det(P) = 0$.

By some calculus, we can find that the condition $C_1 = C_2$ leads to $\det(P) = 0$ and the system uncontrollable.