

**ECE 327: LABORATORY EXERCISE 3:
STATE SPACE REPRESENTATION, CONTROLLABILITY AND OBSERVABILITY**

EXERCISE 1.

Consider a linear plant model described by the differential equation

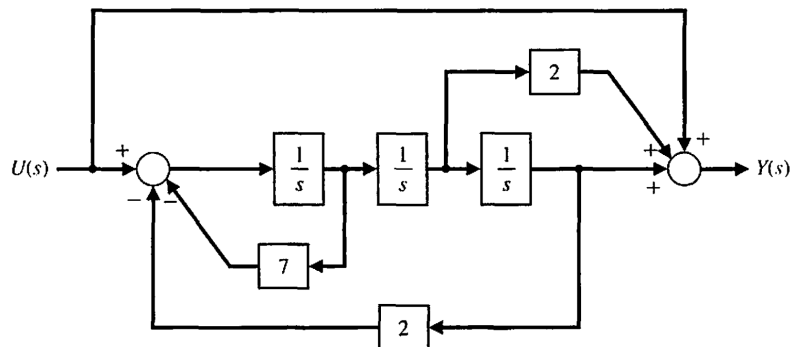
$$\frac{d^3y}{dt^3} + 8\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 4y = 4u(t), \quad (1)$$

where $y(t)$ is the plant output and $u(t)$ is its input. A state model for this system is not unique but depends on the choice of a set of state variables. Based on what we discussed in the class:

- (i) Determine a state-space representation of (1) in phase variable form.
- (ii) Using the “*ss2tf*” function, determine the transfer function $Y(s)/U(s)$.
- (iii) Using the “*lsim*” function, obtain and plot the system response to the initial condition $x(0) = [0 \ -1 \ 1]^T$ for $0 \leq t \leq 10$ if the input is given by:
 - (a) $u(t) =$ unit step function,
 - (b) $u(t) = \sin(2\pi t)$.

EXERCISE 2.

Consider the system shown in block diagram form.



- (i) Obtain a state-space representation of the above system.
- (ii) Using the “*ctrb*” and “*obsv*” functions, determine if the system is controllable and observable.

Exercise 3.

The following model has been proposed to describe the motion of a constant-velocity guided missile:

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -0.1 & -0.5 & 0 & 0 & 0 \\ 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 10 & 0 & 0 \\ 0.5 & 1 & 0 & 0 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} u,$$

$$y = [0 \ 0 \ 0 \ 1 \ 0] \mathbf{x}.$$

- (i) Verify that the system is not controllable by analysing the controllability matrix using the “*ctrb*” function.
- (ii) Develop a controllable state variable model by first computing the transfer function from u to y , then cancel any common factors in the numerator and denominator polynomials of the transfer function. With the modified transfer function just obtained, use the “*tf2ss*” function to determine a modified state variable model for the system.
- (iii) Verify that the modified state variable model in part (ii) is controllable.