ECE327: Laboratory Exercise 6: Pole Placement

Exercise 1. Consider the following state-space system

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u(t)$$

$$y(t) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}.$$

(a) Find the poles of the following system. Is it stable? Plot its step response.
(b) Is the system controllable?
(c) Find a feedback matrix k such that the closed loop system has poles at $-1 \pm 2i$.
(d) Find the new system and plot its step response.

Exercise 2. Consider the following state-space system

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \ddot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{pmatrix} = \begin{pmatrix} -5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ -10.5229 & -1066.67 & -3.38028 & 23.5107 & 0 \\ 0 & 993.804 & 3.125 & -23.5107 & 0 \\ 0 & 0 & 0 & 10 & -10 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \dot{x}_2 \\ x_3 \\ x_4 \end{pmatrix} + \begin{pmatrix} 5 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} u(t)$$

$$y(t) = \begin{pmatrix} 1 & 0 & 0 & 1.223 \times 10^5 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \dot{x}_2 \\ x_3 \\ x_4 \end{pmatrix}.$$

(a) Find the system's eigenvalues. Is it stable?
(b) Examine if the system is controllable. Then, through state feedback, find a matrix K so that the system have 2 poles that approximate a second order system with characteristic polynomial $s^2 + 4s + 11$. Place the remaining 3 poles to a distance of at least 10 times in comparison to the other 2 poles, in the left-half plane of complex numbers.
(c) Find the closed-loop system for matrix K.
(d) Plot the step response of the open and closed-loop system in the same window.